THERMAL DEFORMATION OF THE ACTIVE MEDIUM IN A SOLID-STATE LASER WITH NATURAL COOLING

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An analytical expression is derived for the transient thermal deformation of the active medium in a periodic duty laser with natural cooling.

The effect of thermal conditions on the performance of a solid-state laser is evident in that the emission properties of the active medium change as the temperature rises [1] and that the resonator becomes optically distorted as a result of the thermal deformation of the active medium [2]. In the case of a laser with forced cooling of the active medium, a calculation of the temperature field [3] will yield information about the mean-volume temperature and the amount of thermal deformation. Such a calculation is more difficult in the case of natural cooling, because the heat-transfer processes in the luminaire system, which consists of a pump lamp and the active medium inside a common envelope, have not yet been studied through completely [4-6]. In the first stage of those studies, relations for the mean-volume temperature of the luminaire components have been established and the thermal conductivities have been determined [4], which makes it easier to stipulate the boundary conditions for calculating the temperature field in the active medium. Even in this stage of the study, however, a derivation of analytical relations required substantial simplifications for a formulation of the thermal model.

The initial studies have revealed that thermal deformation of the active medium under conditions of natural cooling produces an optical wedge and is caused by heat transfer from the bulb of the pump lamp. The magnitude of thermal deformation of the active medium β(rad) has been defined [6] according to the relation

$$\beta = \frac{WL}{2R}\Delta\vartheta, \quad W = \frac{\partial n}{\partial\vartheta} + k(n-1). \tag{1}$$

The greatest difficulty arises in determination of the temperature drop $\Delta\vartheta$ between the generatrices of the active medium nearest to and farthest from the lamp. In order to solve this problem, it is necessary to determine the temperature field in the active medium. However, for calculating the temperature field one must stipulate the boundary conditions at the surface of the active medium. This, in turn, requires a detailed study of the heat transfer between the active medium and the other luminaire components. If, after completion of such a study, one could stipulate the boundary conditions in an analytical form, they would obviously only somewhat approximate those in the real process. In the first stage of such studies, therefore, it is worthwhile to treat this problem in its most simplified form and it suffices then to consider only the principal directions of heat flow in the luminaire system. A rather simple expression for the steady-state temperature drop $\Delta\vartheta$ has been obtained [6] which disregards the heat flow in the axial direction, viz.

$$\Delta\vartheta = \frac{P_1}{2\pi\lambda L}\ln\frac{H+R}{H-R}. \tag{2}$$

In the derivation of this relation only the heat transfer between the pump lamp and the active medium was taken into account, not the heat transfer from each of them to the luminaire envelope. Calculations based on expression (2) will, therefore, result in large errors when the pumping power is high and the heat-transfer coefficient from lamp and from active medium to luminaire envelope is correspondingly rather high. We must add here that in the derivation of relation (2) internal heat generation in the active medium was also disregarded, its effect

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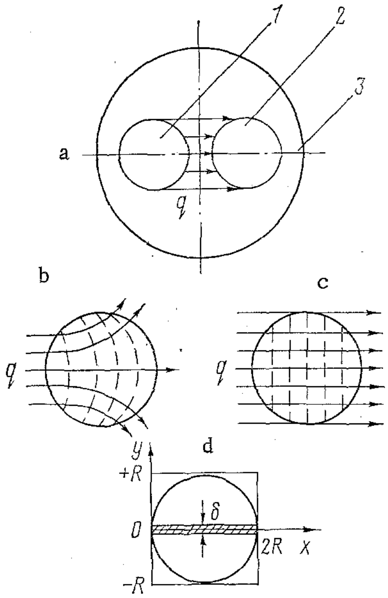

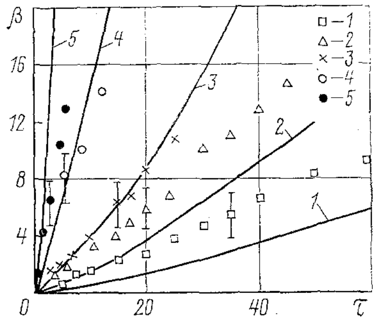

Fig. 1 Fig. 2

Fig. 1. Schematic diagram depicting the propagation of thermal flux from the pump lamp to the active medium: (a) luminaire system (1 — pump lamp; 2 — active medium; 3 — luminaire); (b) passage of thermal flux q (W/m^2) through section of the active medium; (c) simplified schematic diagram depicting the passage of thermal flux q through section of the active medium; (d) conversion from the problem of the temperature field in an infinitely long cylindrical active medium to the problem of the temperature field in an infinitely long plane wall (δ is the thickness of an elementary layer the temperature distribution over which has to be determined); arrows denote flux lines, dash lines denote isotherms.

Fig. 2. Wedgewise thermal deformation β (relative units) as a function of the time τ (relative units) of laser operation in the periodic mode, at various level of pumping power P: 1) 45 W, 2) 75 W, 3) 150 W, 4) 300 W, 5) 750 W; dots denote experimental values, vertical line segments denote the magnitude of experimental error in determination of β.

on the magnitude of $\Delta\vartheta$ not having been determined yet either. These deficiencies make it difficult to use expression (2) for practical estimates even of steady-state temperature drops $\Delta\vartheta$.

The object of this study is to establish approximate analytical relations for the transient temperature drop across a section of the active medium and the wedgewise thermal deformation of the latter, relations which take into account the heat transfer between components of the luminaire system (pump lamp, active medium, luminaire envelope).

We first consider the temperature field in the active medium placed inside the luminaire system. That system is schematically shown in Fig. 1a. Inside the active medium there are heat sources with a specific power Q, a part of the medium surface participates in the heat transfer from the pump lamp, and from another part of the medium surface heat is transferred to the luminaire envelope. The heat transfer between components of the luminaire system is effected through air filling the luminaire cavity.

We make the following assumptions: heat generation in the active medium and in the pump lamp is continuous in time and uniform over the volume, the thermophysical properties of the active medium are isotropic and not temperature-dependent, and the medium separating the components 1, 2, 3, has zero thermal capacity.

We introduce further simplifications, following an analysis of the basic processes in the luminaire system on the basis of the results obtaned by calculation of the mean-surface temperature [4]. It has been demonstrated in study [4] that the luminaire envelope heats up insignificantly and can be regarded as an ambient medium with a slowly rising temperature, until the pump lamp and the active medium reach their steady thermal state. This is a consequence of the large thermal capacity of the luminaire. This feature of the given system of bodies permits us to separately consider the thermal state of the luminaire envelope heated by internal heat generators and by thermal flux from the pump lamp and the active medium, with the temperature of the luminaire envelope $t_3 - t_a = \vartheta_3$ regarded as the ambient temperature. The active medium is heated by internal heat generators and by thermal flux from the pump lamp. Under real conditions the thermal flux from the pump lamp reaches a part of the surface of the active medium, is partly absorbed in the volume of the latter, and is partly dissipated by the opposite other part of the surface. In the general case, moreover, the flux lines can depart from their original direction (Fig. 1b). Since the heat transfer in the luminaire system has not yet been studied thoroughly enough, however, it is difficult to stipulate a law describing the passage of thermal flux through a section of the active medium which will correspond to the real process. We therefore will consider an approximation. We assume that the thermal flux lines from the pump lamp through a section of the active medium are one-directional and the thermal flux density is uniform. We will, moreover, use the simplified scheme of propagation of thermal flux q according to the diagram in Fig. 1c. The validity of this and the further simplifications will become evident from subsequent comparisons of calculations with experimental results. We will recall that, in the final analysis, the sought quantity is not the temperature field in the active medium but the temperature drop across its diameter. It therefore suffices to calculate only the temperature over its diameter within an elementary layer of thickness δ (Fig. 1d). This problem is equivalent to the problem of the temperature field in an infinitely long beam of square cross section (side of the square 2R) with the surfaces y = R, y = −R thermally insulated and the surfaces x = 0, x = 2R participating in heat transfer with the pump lamp and with the luminaire envelope, respectively. The problem thus reduces to calculation of a one-dimensional transient temperature field analogous to the temperature field in an infinitely large plane wall with boundary conditions of the third kind at its surfaces. The ambient temperature at its x = 0 surface is equal to the temperature t_1 of the pump lamp and the ambient temperature at its x = 2R surface is equal to the temperature t_3 of the luminaire envelope. An approximate expression for these temperatures has been obtained in an earlier study [4], viz.,

$$t_1 - t_c = \vartheta_1 = \left(\frac{P_1}{\sigma_{13}} + \vartheta_3\right)[1 - \exp(-m_1\tau)], \quad m_1 = \frac{\sigma_{13}}{C_1}. \tag{3}$$

We will assume that the luminaire temperature rises slowly until $m_1\tau \gtrsim 1$ to $\vartheta_3 \ll P_1/\sigma_{13}$ [4]. Then expression (3) can be rewritten as

$$\vartheta_1 = \frac{P_1}{\sigma_{13}}[1 - \exp(-m_1\tau)]. \tag{4}$$

With all the simplifications made here, the temperature field in the active medium is described by the differential equation

$$\frac{\partial^2\vartheta_2}{\partial x^2} + \frac{Q}{\lambda} = \frac{1}{a}\frac{\partial\vartheta_2}{\partial\tau} \tag{5}$$

with the boundary conditions of the third kind

$$\left.\frac{\partial\vartheta_2}{\partial x}\right|_{x=0} = -\frac{\alpha_{12}}{\lambda}(\vartheta_1 - \vartheta_2|_{x=0}), \tag{6}$$

$$\left.\frac{\partial\vartheta_2}{\partial x}\right|_{x=2R} = \frac{\alpha_{23}}{\lambda}\left.\vartheta_2\right|_{x=2R}. \tag{7}$$

We further assume that at the initial instant of time the temperature is everywhere the same and equal to the ambient temperature

$$t_2|_{\tau=0} - t_c = \vartheta_2|_{\tau=0} = 0. \tag{8}$$

Integration of the system of equations (5)-(8) results in rather unwieldy expressions not very suitable for practical estimates and, for this reason, it is worthwhile to seek an approximate solution to the problem. We therefore average Eq. (5) over the x-coordinate. We introduce the operator

$$I_x[f]=\frac{1}{2R}\int_0^{2R} f dx = f_x, \quad I_x[\vartheta_2(x)]=\vartheta_{2x}.$$

Applying the operator I_x to the first term of Eq. (5) yields

$$I_x\left[\frac{\partial^2\vartheta_2}{\partial x^2}\right]=\frac{1}{2R}\int_0^{2R}\frac{\partial^2\vartheta_2}{\partial x^2}dx=\frac{1}{2R}\int_0^{2R}\partial\left(\frac{\partial\vartheta_2}{\partial x}\right)=\frac{1}{2R}\left.\frac{\partial\vartheta_2}{\partial x}\right|_0^{2R}. \tag{9}$$

Now applying the operator I_x to all terms of Eq. (5) and using Eq. (9), we obtain

$$\frac{1}{a}\frac{\partial\vartheta_{2x}}{\partial\tau}=\frac{Q}{\lambda}+\frac{1}{2R}\left(\left.\frac{\partial\vartheta_2}{\partial x}\right|_{x=2R}-\left.\frac{\partial\vartheta_2}{\partial x}\right|_{x=0}\right).$$

Inserting here the boundary conditions (6) and (7) results in the equation

$$\frac{1}{a}\frac{\partial\vartheta_{2x}}{\partial\tau}=\left[\frac{Q}{\lambda}+\frac{1}{2R\lambda}\left(\alpha_{12}\vartheta_1-\alpha_{12}\vartheta_2\Big|_{x=0}-\alpha_{23}\vartheta_2\Big|_{x=2R}\right)\right]=A, \tag{10}$$

where A denotes the expression inside the square brackets on the right-hand side of Eq. (10).

We make another assumption, viz., that the active medium is at all points of its section heated at approximately the same rate equal to the rate of rise of its mean-volume temperature. Then

$$\frac{1}{a}\frac{\partial\vartheta_2}{\partial\tau}\approx\frac{1}{a}\frac{\partial\vartheta_{2x}}{\partial\tau}=A. \tag{11}$$

A comparison of relations (5) and (11) yields also

$$\frac{\partial^2\vartheta_2}{\partial x^2}\approx A-\frac{Q}{\lambda}. \tag{12}$$

The main difficulty in solving Eq. (10) directly is that it requires expressing the quantities $\vartheta_2|_{x=0}$ and $\vartheta_2|_{x=2R}$ through ϑ_{2x}. According to relation (10), the quantity A in Eqs. (11) and (12) is only a function of time and not a function of the coordinates, which simplifies the integration of Eq. (12)

$$\vartheta_2(\bar{x})=\frac{s_0}{1+s_0+s_2}\left(\frac{s_0+s_2}{s_0}-\bar{x}\right)\vartheta_1+\left[\frac{2s_2+s_0}{2s_0(1+s_0+s_2)}+\frac{2s_2+s_0}{2(1+s_0+s_2)}\bar{x}-\frac{\bar{x}^2}{2}\right]\left(\frac{Q}{\lambda}-A\right)4R^2, \tag{13}$$

where $\bar{x}=x/2R$; $s_0=\alpha_{12}2R/\lambda$; $s_2=\alpha_{12}/\alpha_{23}$.

We will calculate A by determining from expression (13) the mean-volume temperature $t_{2x}-t_a=\vartheta_{2x}$

$$\vartheta_{2x}=\int_0^1\vartheta_2(\bar{x})\,d\bar{x}=\frac{2s_2+s_0}{2(1+s_0+s_2)}\vartheta_1+\frac{(s_0+3s_2)(s_0+4)+s_0s_2}{12s_0(1+s_0+s_2)}\left(\frac{Q}{\lambda}-A\right)4R^2,$$

so that

$$A=\frac{1}{4R^2}\left[\frac{Q4R^2}{\lambda}+\frac{6s_0(2s_2+s_0)}{(s_0+3s_2)(s_0+4)+s_0s_2}\vartheta_1-\frac{12s_0(1+s_0+s_2)}{(s_0+3s_2)(s_0+4)+s_0s_2}\vartheta_{2x}\right]. \tag{14}$$

Inserting expression (14) into expression (11), we obtain the equation

$$\frac{4R^2}{a}\frac{\partial\vartheta_{2x}}{\partial\tau}+\frac{12s_0(1+s_0+s_2)}{(s_0+3s_2)(s_0+4)+s_0s_2}\vartheta_{2x}=\frac{Q4R^2}{\lambda}+\frac{6s_0(2s_2+s_0)}{(s_0+3s_2)(s_0+4)+s_0s_2}\vartheta_1.$$

The solution to this equation, with the initial condition (8) and with expression (4) for ϑ_1, is

$$\vartheta_{2x}(\tau)=\frac{(2s_2+s_0)}{2(1+s_0+s_2)}\frac{P_1}{\sigma_{13}}[1+b\exp(-m_1\tau)-$$

$$-(1+b)\exp(-\tilde{m}_2\tau)] + \frac{(s_0+3s_2)(s_0+4)+s_0s_2}{12s_0(1+s_0+s_2)}\,\frac{Q4R^2}{\lambda}[1-\exp(-\tilde{m}_2\tau)], \tag{15}$$

where

$$b = 1\Big/\left(\frac{m_1}{\tilde{m}_2}-1\right);\quad \tilde{m}_2 = \frac{12(1+s_0+s_2)}{(s_0+3s_2)(s_0+4)+s_0s_2}s_2m_2;\quad m_2 = \frac{\sigma_{23}}{C_2}.$$

Inserting expression (15) into expression (14) and then into expression (13) yields the approximate relation

$$\begin{aligned}\vartheta_2(\bar{x},\tau) &= \frac{s_0}{1+s_0+s_2}\left(\frac{s_0+s_2}{s_0}-\bar{x}\right)\frac{P_1}{\sigma_{13}}[1-\exp(-m_1\tau)] + \\ &+\left[\frac{2s_2+s_0}{2s_0(1+s_0+s_2)}+\frac{2s_2+s_0}{2(1+s_0+s_2)}\bar{x}-\frac{\bar{x}^2}{2}\right]\left\{\frac{Q4R^2}{\lambda}[1-\exp(-\tilde{m}_2\tau)] + \right.\\ &\left.+\frac{6s_0(2s_2+s_0)}{(s_0+3s_2)(s_0+4)+s_0s_2}\,\frac{P_1}{\sigma_{13}}(1+b)[\exp(-m_1\tau)-\exp(-\tilde{m}_2\tau)]\right\}\end{aligned} \tag{16}$$

for the temperature field in the active medium. From relation (16) we obtain the expression

$$\begin{aligned}\Delta\vartheta = \vartheta_2(\bar{x},\tau)|_{\bar{x}=0}-\vartheta_2(\bar{x},\tau)|_{\bar{x}=1} &= \frac{s_0}{1+s_0+s_2}\,\frac{P_1}{\sigma_{13}}\{1-\exp(-m_1\tau) + \\ &+\Omega_1[\exp(-m_1\tau)-\exp(-\tilde{m}_2\tau)]+\Omega_2[1-\exp(-\tilde{m}_2\tau)]\}\end{aligned} \tag{17}$$

for the temperature drop, where

$$\begin{aligned}\Omega_1 &= \frac{3(1-s_2)(s_0+2s_2)}{(s_0+3s_2)(s_0+4)+s_0s_2}(1+b);\\ \Omega_2 &= \frac{1-s_2}{2s_0}\,\frac{Q4R^2}{\lambda}\,\frac{\sigma_{13}}{P_1} = \frac{1-s_2}{2s_2}\,\frac{\sigma_{13}}{\sigma_{12}}\,\frac{P_2}{P_1}.\end{aligned} \tag{18}$$

The results of calculation of the wedgewise transient thermal deformation β in a real laser, according to expressions (17) and (1), are shown in Fig. 2 together with experimental data obtained by the interference method [6]. The values of σ_{ij} and other laser parameters have been taken from our earlier study [4]. For these calculations we used data on the thermo-optical constant W given by other authors [7-9]. The graph in Fig. 2 indicates that the variation of β in time has been calculated with a satisfactory accuracy.

In most cases $P_2 \lesssim 0.1P_1$ and $s_2 \approx 1$ so that, according to the second of expressions (18), $\Omega_2 \simeq 0$ and expression (17) for the steady-state temperature drop $\Delta\vartheta$ can be simplified to

$$\Delta\vartheta = \frac{s_0}{1+s_0+s_2}\,\frac{P_1}{\sigma_{13}} = \frac{P_1}{\sigma_{13}}\,\frac{\sigma}{\lambda L+\sigma},\quad \sigma = \frac{\sigma_{12}\sigma_{23}}{\sigma_{12}+\sigma_{23}}. \tag{19}$$

For very low and very high thermal conductivity λ of the active medium material, expression (19) becomes, respectively,

$$\text{for } \frac{\sigma}{\lambda L} \gg 1 \quad \Delta\vartheta = \frac{P_1}{\sigma_{13}}, \tag{20}$$

$$\text{for } \frac{\sigma}{\lambda L} \ll 1 \quad \Delta\vartheta = \frac{P_1}{\sigma_{13}}\,\frac{\sigma}{\lambda L}. \tag{21}$$

It follows from these two expressions and expressions (1) that in the case (20) minimizing the wedgewise thermal deformation β requires an active medium with the minimum thermooptical constant W. In the case (21) the minimum β will be achieved by using an active medium with the minimum ratio W/λ.

NOTATION

W, thermooptical constant of the active medium; n, refractive index of the active medium; k, thermal expansivity of the active medium; λ, thermal conductivity of the active medium; a, thermal diffusivity of the active medium; L, length of the active medium; 2R, diameter of the active medium; H, distance from axis of the pump lamp to the axis of the active medium; $\vartheta_i = t_i - t_a$, excess temperature; t_i, temperature of the i-th element; t_a, ambient temperature; P_i, power of heat generation within the i-th element; C_i, total thermal capacity of the i-th element; P_{ij}, thermal flux from body i to body j (i, j = 1, 2, 3); σ_{ij}, thermal conductance between body i and body j (i, j = 1, 2, 3); α_{ij}, heat-transfer coefficient from body i to body j (i, j= 1, 2, 3); P, average pump power; and τ, time; 1, pump lamp; 2, active medium; and 3, luminaire envelope.

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